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BRIEF COMMUNICATION

WALL FRICTION FOR TWO-PHASE BUBBLY FLOW IN ROUGH AND SMOOTH TUBES

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1. INTRODUCTION

Many investigations of wall friction and pressure drop for gas–liquid mixture flow inside tubes have been based on the use of the Lockhart–Martinelli parameter (Lockhart & Martinelli 1949), which essentially expresses the ratio of two-phase pressure drop to the pressure drop with only liquid flowing. Inherent in this approach is the supposition that the wall friction and pressure drop are directly related. Whilst this approach is successful for many flow conditions, as discussed by Baroczy (1966), Marié (1987), Martinelli & Nelson (1948), Chisholm (1978), Lockhart & Martinelli (1949) and Chen & Spedding (1981), for example, when the flow undergoes significant acceleration associated with the pressure drop and volumetric expansion of the gas phase, the effects of flow inertia enter the relation between pressure drop and wall friction. This has been considered in detail by Davis (1974) and Herringe & Davis (1978) for gas–liquid mixture flows undergoing significant acceleration along the tube as the critical choking point is approached under conditions of bubbly flow.

Whilst the application of the Colebrook–White equation and of mixing length considerations to two-phase flow have been discussed by Beattie & Whalley (1982), the main body of experimental observations relating to two-phase pressure drop and wall friction have been obtained for flow in smooth tubes. Jensen *et al.* (1985) investigated the effect of relatively large and regular surface protrusions but not the effect of distributed surface roughness. It is the purpose of the work to be described here to investigate the relative influence of pipe roughness on wall friction under compressible two-phase flow conditions. A particular aspect of this is whether the presence of gaseous bubbles in the flow modifies the interaction process between a significantly rough tube wall and the turbulent core flow in the tube, which gives rise to the very large increases of wall friction, due to roughness, which occur in single-phase flow.

The flow regime to be invesigated lies in the homogeneous bubbly flow region of flow classification maps (e.g. Scott 1963; Hewitt & Roberts 1969). The flow velocities were relatively high and some authors apply the term froth flow to describe the regime considered (e.g. Scott 1963). A further aspect of the present investigation relating to the flow regime is the basis on which Re values are calculated. Whilst there is no difficulty as regards the size and mass flux in this regard, the viscosity used has been subject to debate and values ranging from the liquid viscosity to a lower bulk-averaged viscosity have been adopted (Owens 1961; Dukler *et al.* 1964). Since photographic observations for the flows to be discussed showed that bubbles near the tube wall were too large to enter the wall flow sublayers in smooth pipe flow, the liquid viscosity has been adopted. As will be seen, this leads to consistent correlations for the friction factor at different Re values and void fractions.

2. MOMENTUM BALANCE FOR COMPRESSIBLE MIXTURE FLOW

Friction factors were calculated from the observed pressures and flow rates using the equations derived by Davis (1974), which include the effects of mixture compressibility and flow acceleration.

If p_0 is the pressure at the upstream (physically lower) position and p_1 is the pressure at the downstream position, then the mixture density (ρ_m) is given at a point where the pressure is p by the equation

$$\rho_m = \rho_{m0}/[1 - \epsilon_0 + \epsilon_0(p_0/p)], \qquad [1]$$

where ϵ_0 is the mixture void fraction at the reference position (suffix zero). This equation neglects the slip between the phases, which is generally small, and assumes that the gas phase is expanded isothermally as the pressure reduces as a consequence of good thermal contact with the liquid phase and the relatively high specific heat of the liquid phase (Davis 1974). The streamwise momentum balance equation is

$$\tau_w + (d/4)(dp/dx) + (\rho_m u_m d/4)(du_m/dx) + \rho_m g d/4 = 0, \qquad [2]$$

where x denotes the distance along the tube of diameter d, u_m is the mixture velocity, d is the pipe diameter, g is the gravitational acceleration and the wall shear stress is τ_w. The shear stress is related to the mixture conditions by a friction factor f in the usual manner,

$$\tau_w = f\rho_m u_m^2/2. \qquad [3]$$

Combining these equations, [2] can be integrated along the pipe length to obtain the streamwise distribution of pressure. Extending this integration over the test length between positions denoted by suffixes 0 and 1, we then have

$$f(x_1 - x_0)/d = a\{(\bar{p}_1 - 1) + (A/2)\ln[(\bar{p}_1^2 + b\bar{p}_1 + c)/(1 + b + c)] + (B - bA/2)I + C\ln\bar{p}_1\}. \qquad [4]$$

The six constants involved in this equation are expressed only in terms of conditions at the start of the test section (suffix zero):

$$a = (1 - \epsilon_0)/[2D_0(1 - \epsilon_0)^2 + D_0/F_0 f],$$

$$b = 4\epsilon_0 D_0 a,$$

$$c = 2\epsilon_0^2 D_0 a/(1 - \epsilon_0),$$

$$A = \epsilon_0/(1 - \epsilon_0) - b - C,$$

$$B = -\epsilon_0 D_0 - c - C$$

and

$$C = -\epsilon_0^2 D_0/[(1 - \epsilon_0)c],$$

and I represents the result of integration as follows:

$$I = [1/(c - b^2/4)^{1/2}]\{\tan^{-1}[(\bar{p}_1 + b/2)/(c - b^2/4)^{1/2}] - \tan^{-1}[(1 + b/2)/(c - b^2/4)^{1/2}]\}$$

if $c > b^2/4$, or if $b^2/4 > c$, then

$$I = [1/2(b^2/4 - c)^{1/2}]\ln [\![[(b^2/4 - c)^{1/2} - \bar{p}_1 - b/2][(b^2/4 - c)^{1/2} + 1 + b/2]/\{[(b^2/4 - c)^{1/2} + \bar{p}_1 + b/2][(b^2/4 - c)^{1/2} - 1 - b/2]\}]\!].$$

In these expressions $\bar{p}_1 = p_1/p_0$, the pressure ratio across the test length,

$$D_0 = p_{m0} u_{m0}^2/p_0 \quad \text{and} \quad F_0 = U_{m0}^2/gd,$$

where U_{m0} is the average mixture velocity (total volumetric flow divided by pipe area) at the reference section denoted by suffix zero.

If the conditions at the start of the test section are calculated from the pressure and flow rates of the two phases, and if the pressure ratio $\bar{p}_1$ across the test length is known from the observations, then all the constants involved in [4] can be calculated and the friction factor can be determined. Since the friction factor appears in some terms on the r.h.s. of [4], this can be rearranged for the iterative solution, where f_{i+1} and f_i are successive estimates of f in the form

$$2f_{i+1}(x_0 - x_1)D_0(1 - \epsilon_0)^2/d = xD_0/dF_0 + (1 - \epsilon_0)\{(\bar{p}_1 - 1) + (A/2) \times \ln[(\bar{p}_1^2 + b\bar{p}_1 + c)/(1 + b + c)] + (B - bA/2)I + C\ln\bar{p}_1\}, \qquad [5]$$

where f_i is used in the calculation of all terms on the r.h.s. of this equation. This iterative process converged to within 0.1% of the limit within between 5 and 30 iterations, the iteration being slower for low mixture speeds and smaller pressure drop. The condition of $b^2 < 4c$ was found to apply for the majority of flows, except those having high flow rate and pressure drop where gravitational effects were not as strong.

3. EXPERIMENTAL OBSERVATIONS OF FRICTIONAL PRESSURE DROP

The experimental observations were carried out in a vertical tube containing upward flows of air–water mixtures. The tube was of 51 mm dia, and the mixture was formed in a conical mixing chamber of the type described by Herringe & Davis (1976). The mixing chamber was of overall length 0.405 m, and was in the form of a smooth conical contraction from the base plate of dia 0.102 m to the outlet of dia 0.051 m. Air was injected through a central hole of dia 0.013 m in the base plate, and water through eight 0.013 m holes in the base plate on a 0.076 m dia circle. As described by Herringe & Davis (1976), this type of mixer gives a well-homogenized bubbly mixture flow at its outlet. The flows remained relatively steady and well-mixed bubbly flows without any tendencies to form slugs or to break down into annular patterns. Experiments were carried out using smooth Perspex tubes, the test length between pressure measurement positions being 4.3 m, with a 1.1 m settling length between the mixer and test sections. The flows discharged to the atmosphere via a U-bend at the top outlet. One test length of tube was artificially roughened by coating it with epoxy resin internally and embedding sand particles in this bonding layer. This reduced the diameter presented to the flow slightly, but it was found that a relatively uniform layer of this roughening treatment could be obtained by rotating the tube during application. The sand particles were graded with average dia 1.0 mm. Because this strongly roughened tube did not have a well-defined roughness-to-diameter ratio, comparative frictional pressure drop measurements were made with a single-phase water-only flow to provide a point of reference as regards the influence of the pipe roughness on the wall friction factors. Pressure differences across the test section were measured using precision Bourden gauges with an accuracy of 300 Pa. Care was taken to ensure that the connecting leads were maintained full of water so that the actual pressures at the different vertical positions on the test section could be calculated accurately. Flow rates of the two phases were measured externally by means of standard orifice meters in the two supply lines. Test flows had velocities ranging from 0.8 to 5.5 ms^{-1} and void fractions between 0.1 and 0.45 in the bubbly regime. Untreated tap water was used in the experiments. Extensive and detailed flow structure observations have been reported elsewhere by Herringe & Davis (1976), and the flows in the present tests all appeared visually to be of the bubbly type. The corresponding range of the quality of the mixture in the present tests was between 10^{-4} and 10^{-3}, depending on the particular void fraction and pressure of the flow.

The friction factors determined from a series of approx. 300 tests in the smooth and rough tubes are shown in figures 1a and 1b. The results for liquid-only flow can be seen to lie very close to the Colebrook–White equation for the smooth pipe tested (Colebrook 1939):

$$1/\sqrt{f} = 2\log_{10}(d/\varepsilon) - 2\log_{10}[1 + (9.28d/\mathrm{Re}\sqrt{f}\varepsilon)], \qquad [6]$$

where ε is the pipe roughness and Re is the Reynolds number.

For the rough pipe it was found that a least-squares minimization yielded an effective pipe roughess of 1.3 mm in the Colebrook–White equation. This is in quite close accord with the nominal 1 mm graded sand used in producing the rough wall finish, and has resulted in an increase of the wall friction factor to about four times the smooth pipe value. Figures 1a and 1b also show the friction factors observed under two-phase conditions, and it can be seen that these are generally higher than those under single-phase conditions. The difference is larger for the smooth wall pipe, and increases sharply as the Re reduces below a value of 150,000. For the rough wall pipe a similar but relatively smaller increase in the two-phase friction factors is observed for decreasing Re. At the highest Re tested (170,000 in these experiments), it can be seen that the two-phase and single-phase fricton factors are virtually identical in both the smooth and rough wall cases, and it appears therefore that the increase in the friction factor due to roughness is then almost identical for single- and two-phase flows. This result indicates that the wall roughness acts in a

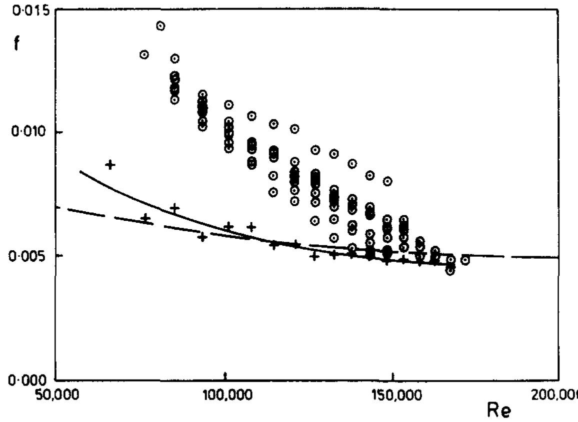

Fig. 1a. Variation of the friction factor with Re for the smooth tube: +, liquid-only observations; ○, gas–liquid mixture flows; ———, Colebrook–White equation (smooth pipe).

similar manner in both cases, and that a similar correlation between friction factor and Re can be applied at higher Re for single- and two-phase flows. Re has been defined here in terms of the mixture density $[\rho_m = \rho_G\epsilon + \rho_L(1-\epsilon)]$ and the liquid viscosity, and from the foregoing discussion it appears therefore that there is only limited interaction between the pipe roughness and the two-phase mixture in terms of friction, and that the roughness primarily interacts with the liquid-only layer immediately adjacent to the pipe wall in respect of friction.

The friction factor was found to increase systematically with mixture void fraction under both smooth and rough wall conditions. This is illustrated in figure 2, where results at the various discrete Re values tested are shown. For the smooth wall pipe the increase is relatively strong and uniform over the range of Re tested, whilst for the rough wall tube the increases are relatively smaller—although comprising similar absolute increases in the friction factor. It also appears from figure 2 that there is a reduced sensitivity of the friction factor to the mixture void fraction under rough wall conditions at the higher Re values. This effect is, however, influenced by the overall operating constraints of the test system as these latter results also correspond to a lower range of void fractions. Application of a linear regression to the friction factor results in terms of the void fraction at various Re gives rise to the variation of the friction factor with void fraction and Re shown in figure 3. These results indicate that the increase in the friction factor with void fraction is confined to a higher range of void fractions as the Re increases. Further, for the smooth wall tube this development appears to occur in a higher range of Re. In some respects these effects are generally similar to the variation of single-phase friction factors, where significant variations of the friction factor with Re extend to higher Re for smooth wall tubes, and it could be concluded that the proportionately smaller influence of the mixture void fraction for the rough wall tube is, effectively,

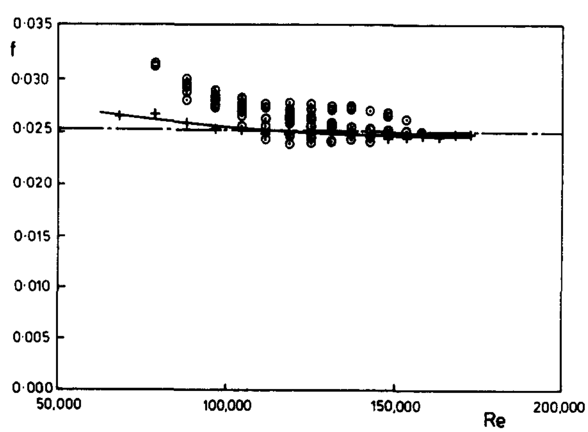

Fig. 1b. Variation of the friction factor with Re for the rough tube: +, liquid-only observations; ○, gas–liquid mixture flows; —·—, Colebrook–White equation ($\varepsilon/d = 0.026$).

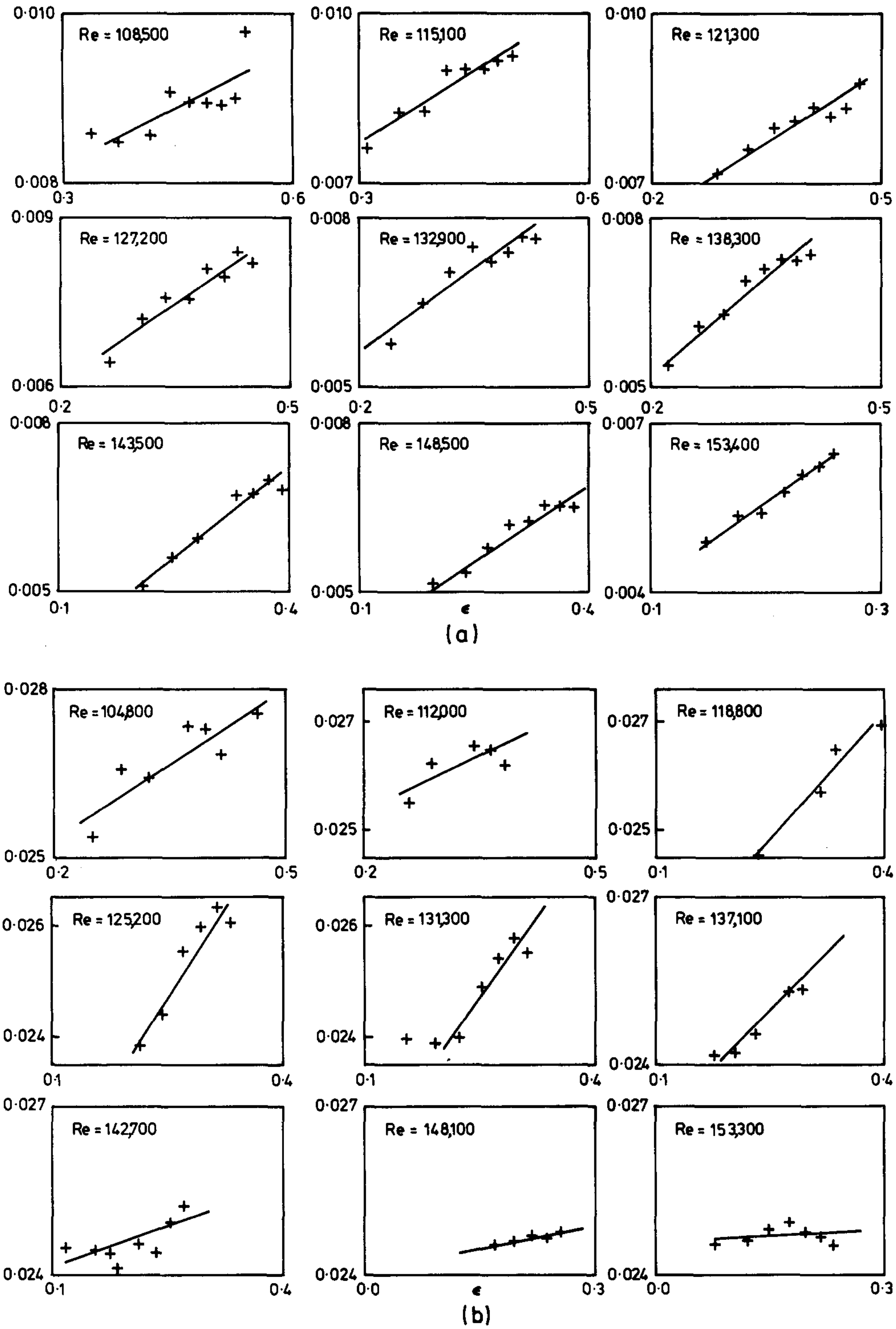

Fig. 2. Increase of the friction factor with the mixture void fraction: (a) smooth tube; (b) rough tube.

a reflection of the similar tendency for the friction factor to vary less for single-phase flow in rough wall tubes.

In order to resolve some general aspects of the influence of the gas component of the flow on the wall friction, high-speed flash photographs were taken through the tube wall using an E.G. & G. Microflash which produced a flash of $<1\ \mu s$ duration. This was combined with a close-up camera and high-speed film to photograph bubbles nearest the wall and to thereby measure the

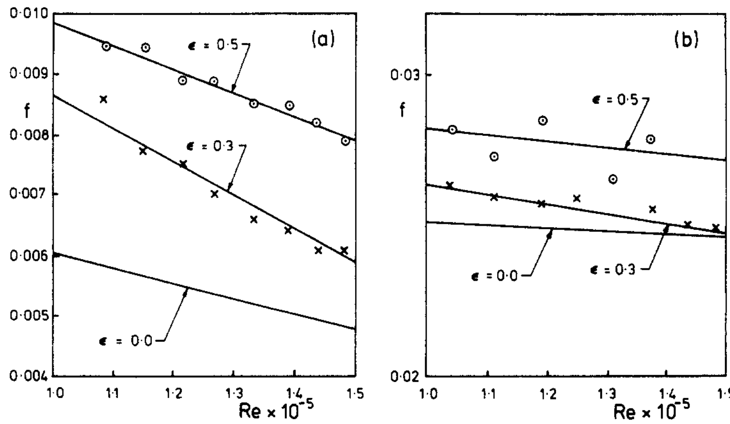

Fig. 3. Variation of the friction factor with Re and the void fraction: (a) smooth tube; (b) rough tube; ——, linear regression fit to the data.

minimum bubble size. A special window was inserted in the rough wall pipe for this purpose. It was found that the smallest bubbles for smooth wall conditions were 0.28 mm dia, whilst under rough wall conditions this reduced to 0.06 mm approx. Much more detailed investigations of the flow in the smooth pipe have been reported by Herringe & Davis (1976), who showed that the mean bubble size in the turbulent core of the pipe flow could be estimated from the interaction of turbulent mixing and surface tension interfacial energies. The smallest bubble sizes observed here photographically at the wall are appreciably less than the mean bubble size in the pipe, as can be seen from the bubble size distributions observed by Herringe & Davis (1976). It is not considered that the present observations of reduced bubble size near the wall due to roughness can be interpreted as indicating substantial changes in overall energies due to roughness, as the observed size reductions only apply in the thin layer immediately adjacent to the wall. From these observations it is clear that the roughness on the tube wall does interact with the gas component of the flow to reduce the minimum bubble size near the wall, and this therefore provides physical evidence of the interaction of the wall with the two-phase flow. Calculations of the thickness (δ_b) that would be expected for the turbulent buffer layer in the flow can be based on the approximate relation for pipe flows,

$$\delta_b/d = 70/\mathrm{Re}\sqrt{f}.$$

This yields a turbulent buffer layer thickness of 0.36 mm for a typical smooth tube flow and 0.18 mm for a typical rough tube flow. On this basis, it is therefore to be expected that the smallest bubbles in the flow can only just enter and influence the turbulent buffer layer near the tube wall, and thereby influence the wall friction process to the moderate extent that has been observed in terms of the increase of friction with void fraction.

4. CONCLUSION

The experiments with rough and smooth pipes have shown generally similar increases in the wall friction factor due to roughness for both single- and two-phase flows. However, flow in the rough wall tube showed a somewhat weaker dependence on the mixture gas content and this dependence was confined to a lower Re range for the rough wall tube. Minimum bubble sizes were less than the turbulent buffer layer thickness for both rough and smooth wall tubes, and it is therefore to be expected that the wall friction would be modified to some extent by the gas phase, as has been found to be the case. Thus, the bubbly gas content of the flow has a limited but systematic effect on the friction coefficient in both rough and smooth tubes. Generally, comparable absolute increases in the friction factor occurred in rough and smooth tubes, giving rise to a higher proportionate increase in the friction factor in the latter case. Correlations of data based on the liquid viscosity give consistent results for both smooth and rough wall cases for the bubbly flows investigated.

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